ISSN:0975-9646

Manju Pandey et al, / (IJCSIT) International Journal of Computer Science and Information Technologies, Vol. 3 (2) , 2012,3775-3777

New Aggregation Operator for Triangular Fuzzy Numbers based on the Arithmetic Means of the Slopes of the L- and R- Membership Functions

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***Abstract*— In recent work authors have proposed four new aggregation operators based on the arithmetic and geometric means of the L- and R- or right side and left side apex angles for triangular and trapezoidal fuzzy numbers respectively. In this paper authors propose a new aggregation operator for TFNs in which the L- and R- membership function lines of the aggregate TFN have slopes which are the arithmetic means of the corresponding L- and R- slopes of the individual TFNs The L- and R- membership function lines are treated independently. Computation of the aggregate is demonstrated with a numerical example. Corresponding arithmetic and geometric aggregates as well as results from the recent work of the authors on TFN aggregates have also been computed.**

Keywords*— LR Fuzzy Number, Triangular Fuzzy Number, Apex Angle, Slope, L- and R- Membership Functions, Aggregation Operator, Arithmetic and Geometric Mean

I. Introduction

Many different types of fuzzy numbers are defined in the literature dealing with fuzzy logic and applications. In this paper only one class of fuzzy numbers i.e., Triangular Fuzzy Numbers (TFNs) which are a special class of LR fuzzy numbers are treated.

A. *Triangular Fuzzy Numbers*

TFNs are extensively used in fuzzy applications owing to their simplicity. TFNs are used in fuzzy applications where uncertainty exists on both sides of a value or parameter [1]. TFNs are characterized by an ordered triplet of real numbers <l,m,u>. Figure depicts a TFN (l,m,u) with values v on x-axis and membership or grade μ along y-axis. The L- and R- apex angles are shown with arcs having one and two dashes respectively. Vertices of the triangle are at (l, 0), (m, 1) & (u,0) moving clockwise. Line between (l, 0) and (m, 1) and between (m, 1) and (u,0) are the membership functions for the values in the intervals [l, m] and [m, u] respectively. Membership function of the TFN is the piecewise continuous linear function represented by the lines L and R respectively. Intuitive meaning of such TFN is that the fuzzy number is approximately m [2]. The value of the fuzzy number varies in the range l and u. The possibility or membership grade of the number being a specified value v between l and u, v ∈ [l, u] is represented by the ordinate of the projection of v on L or R accordingly as l ≤ v ≤ m or m ≤ v ≤ u. [l,m] is the range or interval which depicts or represents the possibilities of the number being a specific value less than m and [m, u] is the interval which depicts the possibilities of the number being a specific value greater than m. Possibility takes on the maximum value of 1 at m and reduces with increasing distance on either side (left / right) of m, becoming zero beyond l at the left and m at the right respectively.

Mathematically, a TFN is defined as follows. Let l, m, $u \in \mathbf{R}$, $l < m < u$. The fuzzy number t: $\mathbf{R} \rightarrow [0, 1]$ denoted by

$$t = \begin{cases} 0, & if\ x < l \\ \dfrac{x-l}{m-l}, & if\ l \le x \le m \\ \dfrac{u-x}{u-m}, & if\ m \le x \le u \\ 0, & if\ x > u \end{cases}$$

is called a triangular fuzzy number [1][2][3][4].

II. Fuzzy Aggregation

Aggregation operations on fuzzy numbers are operations by which several fuzzy numbers are combined to produce a single fuzzy number [5]. An excellent account of Mathematical Aggregation Operators is given in [6]

A. *Arithmetic Mean*

The arithmetic mean aggregation operator [5][7] defined on n TFNs $<l_1,m_1,u_1>$, $<l_2,m_2,u_2>$, …, $<l_i,m_i,u_i>$, …,$<l_n,m_n,u_n>$ produces the result $<\bar{l}, \bar{m}, \bar{u}>$ where

$$\bar{l} = \frac{1}{n}\sum_{1}^{n} l_i\ ,\ \bar{m} = \frac{1}{n}\sum_{1}^{n} m_i\ ,\ \text{and}\ \bar{u} = \frac{1}{n}\sum_{1}^{n} u_i$$

B. *Geometric Mean*

The geometric mean aggregation operator [5][8] defined on n TFNs $<l_1,m_1,u_1>$, $<l_2,m_2,u_2>$, …, $<l_i,m_i,u_i>$, …, $<l_n,m_n,u_n>$ produces the result $<\bar{l}, \bar{m}, \bar{u}>$ where

$$\bar{l} = \left(\prod_{1}^{n} l_i\right)^{\frac{1}{n}},\ \bar{m} = \left(\prod_{1}^{n} m_i\right)^{\frac{1}{n}},\ \text{and}\ \bar{u} = \left(\prod_{1}^{n} u_i\right)^{\frac{1}{n}}$$

Other aggregation operators have also been defined in literature. For examples see [8][9][10][11].

C. Applications of Aggregation

The combination/aggregation/fusion of information from different sources is at the core of knowledge based systems. Applications include decision making, subjective quality evaluation, information integration, multi-sensor data fusion, image processing, pattern recognition, computational intelligence etc. An application of aggregation operators in fuzzy multicriteria decision making is discussed in [7][8]. Another application in sensor data fusion is discussed in [10].

D. Organization of the Paper

In this paper a new aggregation operator for TFNs based on arithmetic means of the slopes is proposed. The L- and R-membership functions have been treated independently. The operator is described in the next section. Computation of the aggregate is demonstrated with a numerical example. Corresponding arithmetic and geometric aggregates as well as results from the recent work of the authors on TFN aggregates have also been computed..

III. PROPOSED AGGREGATION OPERATOR

Consider the TFN shown in Fig. 1. The most likely value of this TFN is m where the possibility μ= 1. The slope of L-membership line is tan(£mlp) while the slope of the R-membership line is tan(£mux) = -tan(£pum)

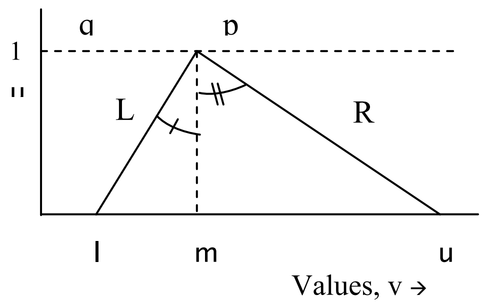

FIG.1: TRIANGULAR FUZZY NUMBER

Averaging the L-slopes over n TFNs we have

$$\frac{1}{n}\sum_{1}^{n} tan(\pounds mlp)_i = \frac{1}{n}\sum_{1}^{n}\frac{1}{(m_i - l_i)}$$

This represents the slope of the resultant aggregate left line $\bar{L}$.

Similarly, it can be shown that the slope of the resultant fuzzy aggregate right line $\bar{R}$ is $-\frac{1}{n}\sum_{1}^{n}\frac{1}{(u_i - m_i)}$..

Under identical treatment, it can be shown that $\bar{m} = \frac{1}{n}\sum_{1}^{n} m_i$.

Subsequently it can be shown that

$$\bar{l} = \frac{1}{n}\sum_{1}^{n} m_i - \frac{n}{\left(\sum_{1}^{n}\frac{1}{(m_i - l_i)}\right)}$$, and

$$\bar{u} = \frac{1}{n}\sum_{1}^{n} m_i + \frac{n}{\left(\sum_{1}^{n}\frac{1}{(u_i - m_i)}\right)}$$

IV. NUMERICAL EXAMPLE

Consider the two triangular fuzzy numbers **<1,1.5,3>** and **<5,6.5,9>** aggregate TFN is computed as

$\bar{m}$=4; $\bar{l} = 4 - \frac{2}{\left(\frac{1}{1.5-1} + \frac{1}{6.5-5}\right)}$ =3.25; Similarly, $\bar{u}$ can be computed as 5.875.

We have the aggregate as **<3.25,4,5.875>.** The arithmetic mean aggregate is **<3,4,6>** and the geometric mean aggregate is **<2.24,3.12,5.20>** respectively. We have the aggregate as **<3.117,4,5.90>** considering arithmetic means of apex angles and **<2.3219,3.1224,5.00>** considering geometric mean of apex angles.

V. CONCLUSIONS

In this paper we have defined a new aggregate of triangular fuzzy numbers based on the arithmetic mean of the slopes. The L- and R- membership lines and their slopes have been treated independently. A numerical example has been worked out. The aggregate is the resultant piecewise continuous membership function line whose L- and R- slopes are the arithmetic means of the L- and R- slopes of the individual TFNs. The suitability of the aggregation operator proposed in this paper in different fuzzy logic applications involving aggregation remains to be explored.

ACKNOWLEDGEMENTS

Manju Pandey gratefully acknowledges Shri Harendra Bikrol, Associate Professor, and Shri Mithilesh Atulkar, Associate Professor and Head of Computer Applications Department at National Institute of Technology (NIT), Raipur, Chhattisgarh, India, for their constant help and support. She is also thankful to Dr. Shrish Verma, Head of Electronics and Information Technology Departments and Central Computer Centre at NIT Raipur for his encouragement and support.

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